Adaptive Inverse Control
We dedicate this work to our families over the generations. They helped us and inspired us.

I would like to dedicate this book to my grandsons Jeffrey and Adam Sklarin, to their parents Rick and Debbie, to my daughter Leslie, to my wife Ronna Lee, and to the memory of my parents Moe and Ida Widrow.

Bernard Widrow

I would like to dedicate this book to my son Elad, to my daughter Algith, to my wife Rina, to my mother Sarah and to the memory of my father Benjamin Walach.

Eugene Walach
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A Special Dedication to the Memory of Derrick Nguyen

Derrick Nguyen completed the Ph.D. in Electrical Engineering at Stanford University in June 1991. He was the first to develop neural controls for the “truck backer-upper,” based on backpropagation through time. His work has wide application in the field of nonlinear control. In his short life he accomplished a great deal. He was a favorite of all who knew him.
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Preface

In this book, methods of adaptive signal processing are borrowed from the field of digital signal processing to solve problems in dynamic systems control. Adaptive filters, whose design and behavioral characteristics are well known in the signal processing world, can be used to control plant dynamics and to minimize the effects of plant disturbance. Plant dynamic control and plant disturbance control are treated herein as two separate problems. Optimal least squares methods are developed for these problems, methods that do not interfere with each other. Thus, dynamic control and disturbance cancelling can be optimized without one process compromising the other. Better control performance is the result. This is not always the case with existing control techniques.

Inverse control of plant dynamics involves feed-forward compensation, driving the plant with a filter whose transfer function is the inverse of that of the plant. Inverse compensation is well known in signal processing and communications.

Every MODEM in the world uses adaptive filters for channel equalization. Similar techniques are described here for plant dynamic control. Inverse control is feed-forward control. The same precision of feedback that is obtained with existing control techniques is also obtained with adaptive feed-forward control since feedback is incorporated in the adaptive algorithm for obtaining the parameters of the feed-forward compensator.

Inverse control can be used effectively with minimum phase and non-minimum phase plants. It cannot work with unstable plants, however. They must first be stabilized with conventional feedback, of any design that simply achieves stability. Then the plant and stabilizing feedback can be treated as an equivalent stable plant that can be controlled in the usual way with adaptive inverse control. Model reference control can be readily incorporated into adaptive inverse control.

Adaptive noise cancelling techniques are described that allow optimal reduction of plant disturbance, in the least squares sense. Adaptive noise cancelling does not affect inverse control of plant dynamics. Inverse control of plant dynamics does not affect adaptive disturbance cancelling. If initial feedback is needed to provide plant stabilization, the design of the stabilizer has no effect on the optimality of the adaptive disturbance canceller.

The designs of the adaptive inverse controller and of the adaptive disturbance canceller are quite simple once the control engineer gains a mastery of adaptive signal processing. This book provides an introductory presentation of this subject with enough detail to do system design. The mathematics is simple and indeed the whole concept is simple and easy to implement, especially when compared with the complexity of current control methods.

Adaptive inverse control is not only simple, but it affords new control capabilities that can often be superior to those of conventional systems. Many practical examples and applications are shown in the text.

Another feature of adaptive inverse control is that the same methods can be applied to adaptive control of nonlinear plants. This is surprising because nonlinear plants do not have transfer functions. But approximate inverses are possible. Experimental results with nonlinear plants have shown great promise. Optimality cannot be proven yet, but excellent
results have been obtained. This is a very promising subject for research. The whole area
of nonlinear adaptive filtering is a fascinating research field that already shows great re-
sults and great promise.

This book was originally published under the title *Adaptive Inverse Control*. We are
grateful to IEEE Press and John Wiley, Inc. for bringing it back into print. We are also
grateful to colleagues Gene Franklin, Karl Johan Åström, Jose Cruz, Brian Anderson,
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preparation of the drawings and final manuscript.

BERNARD WIDROW
*Stanford, California*

EUGENE WALACH
*Haifa, Israel*
Chapter 1

The Adaptive Inverse Control Concept

1.0 INTRODUCTION

Adaptive filtering techniques have been successfully applied to adaptive antenna systems [1–20]; to communications problems such as channel equalization [21–30] and echo cancelation in long-distance telephony [31–39]; to interference canceling [40–46]; to spectral estimation [47–57]; to speech analysis and synthesis [58–60]; and to many other signal processing problems. It is the purpose of this book to show how adaptive filtering algorithms can be used to achieve adaptive control of unknown and possibly time varying systems.

The system to be controlled, usually called the "plant," may be noisy, that is, subject to disturbances, and for the most part it may be unknown in character. The plant and its internal disturbances may be time variable in an unknown way. In some cases, the plant might even be unstable. Adaptive control systems for such plants would be advantageous over fixed systems since the parameters of adaptive systems can be adjusted or tailored to the unknown and varying requirements of the plant to be controlled. Adaptivity finds a natural area of application in the control field [88].

In the past two decades or so, many hundreds of papers have been published on adaptive control systems in the Transactions of the IEEE Control Systems Society, in Automatica, in the IFAC (International Federation for Automatic Control) journals and conference proceedings, and elsewhere. At the same time, a very large number of papers on adaptive signal processing and adaptive array processing have appeared in the Transactions of the IEEE Signal Processing Society, Antennas and Propagation Society, Communications Society, Circuits and Systems Society, Aerospace and Electronics Society, the Proceedings of the IEEE, and elsewhere. Many books have been published on these subjects. The two schools of thought, adaptive controls and adaptive signal processing, have developed almost independently. The control theorists have by and large studied adaptive control using

1 Some prior knowledge of the character of the plant and its internal disturbances will be needed in order to establish proper control. For example, at least a rough idea of the transient response time of the plant would be required in order to model it adaptively. Some idea of how rapidly the plant characteristics change for plants that vary over time would be needed. Some knowledge of the plant disturbance would be useful, such as disturbance power level at the plant output. Detailed knowledge of the plant and its disturbances would not be required however.